7. Y. I. Nissim, A. Lietoila, R. B. Gold, and J. F. Gibbons, J. Appl. Phys., 50, No. 1, 274-279 (1979).
8. M. L. Bürgener and R. E. Reedy, J. Appl. Phys., 53, No. 6, 4357-4363 (1982).
9. I. D. Calden and R. Sue, J. Appl. Phys., 53, No. 11, 7545-7550 (1982).
10. D. M. Kim, R. R. Shah, and D. L. Cristhwait, J. Appl. Phys., 51, No. 6, 3121-3125 (1980).
11. J. E. Mody and R. H. Hendel, J. Appl. Phys., 53, No. 6, 4364-4371 (1982).
12. S. A. Kokorowski, G. L. Olsen and L. D. Hess, Laser and Electron Beam Solid Interaction and Materials Processing, N. Y. (1981), pp. 139-145.
13. W. H. Hawkins and D. K. Biegelsen, Appl. Phys. Lett., 42, No. 4, 358-360 (1983).
14. M. Born and E. Wolf, Principles of Optics, Pergamon (1970).
15. C. Y. Ho, R. W. Powell, and P. E. Liley, J. Phys. Chem, 3, Suppl. 1, 558-590 (1974).
16. M. Kimura and Y. Egami, Appl. Phys. Lett., 44, No. 4, 420-422 (1984).

SOME FORMULAS OF OPERATIONAL CALCULUS FOR STEP FUNCTIONS GENERATED BY SPECIAL FUNCTIONS

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UDC 517.942.82:536.24

One-dimensional Laplace transforms of some step functions are presented.

Formulas of operational calculus for step functions play an important role in discrete analysis, in particular, in the solution of various equations [1, 2]. In the present paper, which is a continuation of [3], we present a table of new operational relationships for step functions that are special functions of the function $[t]$, the integer part of t. The relationships are presented in two columns: the function $f([t])$ in the left hand column and its Laplace transform F(p) in the right hand column, where

$$F(p)=\int_0^\infty f([t])\exp(-pt)\,dt=\frac{1-e^{-p}}{p}\sum_{k=0}^{\infty} f(k)\,e^{-kp}.$$

Here $f([t]) = f(k)$ for $k \le t < k + 1$, $k = 0, 1, 2, \ldots$; $\mathrm{Re}\, p > 0$, unless the contrary is indicated. The notation employed is that commonly used in the mathematical literature (see, for example, [4-7]).

TABLE 1. Laplace Transforms of Some Step Functions

No.	$f([t])$	$F(p)$
1	$x^{[t]}\Gamma([t],\ a)$	$\frac{1-e^{-p}}{p}\left[\exp(xe^{-p})\,\mathrm{Ei}\left(-a+\frac{e^{p}}{x}\right)-\mathrm{Ei}(-a)\right]$, $\lvert axe^{-p}\rvert<1$
2	$\frac{x^{[t]}}{[t]!}\Gamma([t],\ a)$	$\frac{1-e^{-p}}{p}\mathrm{Ei}(axe^{-p}-a)$, $a(1-xe^{-p})>0$
3	$x^{[t]+1}\zeta([t]+2)$	$\frac{1-e^{p}}{p}[\mathrm{C}+\psi(1-xe^{-p})]$, $\lvert xe^{-p}\rvert<1$
4	$x^{2[t]}\zeta(2[t])$	$\frac{\pi x}{2}\,\frac{e^{-3p/2}-e^{-p/2}}{p}\,\mathrm{ctg}(\pi xe^{-p/2})$, $\lvert xe^{-p/2}\rvert<1$

Computing Center, Academy of Sciences of the USSR, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 53, No. 6, pp. 1010-1014, December, 1987. Original article submitted November 11, 1986.

 0022-0841/87/5306-1472$12.50

TABLE 1 (continued)...

No.	$f([t])$	$F(p)$
5	$\frac{x^{[t]+2}}{[t]+2}\zeta([t]+2)$	$\frac{1-e^{-p}}{p}e^{2p}[\ln\Gamma(1-xe^{-p})-Cxe^{-p}],\quad \lvert xe^{-p}\rvert<1$
6	$\frac{x^{2[t]+2}}{[t]+1}\zeta(2[t]+2)$	$\frac{e^{p}-1}{p}\ln[\pi xe^{-p/2}\operatorname{cosec}(\pi xe^{-p/2})],\quad \lvert xe^{-p/2}\rvert<1$
7	$\frac{(a)_{[t]}}{[t]!}x^{[t]}\zeta([t]+a)$	$\frac{1-e^{-p}}{p}\zeta(a,\ 1-xe^{-p}),\quad \lvert xe^{-p}\rvert<1$
8	$\binom{n+[t]-1}{[t]}x^{[t]}\zeta(n+[t])$	$\frac{(-1)^{n}}{(n-1)!}\,\frac{1-e^{-p}}{p}\psi^{(n-1)}(1-xe^{-p}),\quad \lvert xe^{-p}\rvert<1$
9	$\frac{x^{[t]}}{[t]!}D_{[t]+\nu}(a)$	$\frac{1-e^{-p}}{p}\exp\left[\frac{x}{4}e^{-p}(2a-xe^{-p})\right]D_{\nu}(a-xe^{-p})$
10	$\frac{(-\nu)_{[t]}}{[t]!}x^{[t]}D_{\nu-[t]}(a)$	$\frac{1-e^{-p}}{p}\exp\left[\frac{x}{4}e^{-p}(xe^{-p}-2a)\right]D_{\nu}(a-xe^{-p})$
11	$\frac{x^{[t]}}{[t]!}J_{\nu\pm[t]}(a)$	$\frac{1-e^{-p}}{p}\left(1\mp\frac{2x}{a}e^{-p}\right)^{\mp\nu/2}J_{\nu}(\sqrt{a^{2}\mp 2axe^{-p}}),$ $2\lvert xe^{-p}\rvert<a$
12	$\frac{x^{[t]}}{[t]!}\cdot I_{\nu+[t]}(a)$	$\frac{1-e^{-p}}{p}\left(1+\frac{2x}{a}e^{-p}\right)^{-\nu/2}I_{\nu}(\sqrt{a^{2}+2axe^{-p}})$
13	$\frac{x^{[t]}}{[t]!}I_{\nu-[t]}(a)$	$\frac{1-e^{-p}}{p}\left(1+\frac{2x}{a}e^{-p}\right)^{\nu/2}I_{\nu}(\sqrt{a^{2}+2axe^{-p}}),$ $2\lvert xe^{-p}\rvert<a$
14	$x^{[t]}P_{[t]}(a)$	$\frac{1-e^{-p}}{p(1-2axe^{-p}+x^{2}e^{-2p})^{1/2}},\quad \lvert xe^{-p}\rvert<\min\lvert a\pm\sqrt{a^{2}-1}\rvert$
15	$\frac{x^{[t]+1}}{[t]+1}P_{[t]+1}(a)$	$\frac{e^{p}-1}{p}\ln\{2[1-axe^{-p}+(1-2axe^{-p}+$ $+x^{2}e^{-2p})^{1/2}]^{-1}\},\quad \lvert a\rvert<1$
16	$\frac{x^{[t]}}{[t]!}P_{[t]}(a)$	$\frac{1-e^{-p}}{p}\exp(axe^{-p})J_{0}(xe^{-p}\sqrt{1-a^{2}}),\quad \lvert a\rvert<1$
17	$\frac{x^{[t]}}{([t]!)^{2}}P_{[t]}(a)$	$\frac{1-e^{-p}}{p}I_{0}(2xe^{-p}\sqrt{a-1})\,I_{0}(2xe^{-p}\sqrt{a+1})$
18	$x^{[t]}L_{[t]}^{\alpha}(a)$	$\frac{1-e^{-p}}{p}(1-xe^{-p})^{-\alpha-1}\exp\left(\frac{ax}{x-e^{p}}\right),\quad \lvert xe^{-p}\rvert<1$
19	$\frac{x^{[t]}}{[t]+\alpha}L_{[t]}^{\alpha}(a)$	$\frac{1-e^{-p}}{p}(axe^{-p})^{-\alpha}\gamma\left(\alpha,\ \frac{ax}{e^{p}-x}\right),\quad \operatorname{Re}\alpha>0,\ \lvert xe^{-p}\rvert<1$
20	$\frac{x^{[t]}}{(\alpha+1)_{[t]}}L_{[t]}^{\alpha}(a)$	$\Gamma(\alpha+1)\frac{1-e^{-p}}{p}(axe^{-p})^{-\alpha/2}\exp(xe^{-p})J_{\alpha}(2\sqrt{axe^{-p}})$
21	$x^{[t]}L_{[t]}(a)L_{[t]}(b)$	$\frac{1-e^{-p}}{p}(1-xe^{-p})^{-1}\exp\left(x\frac{a+b}{x-e^{p}}\right)I_{0}\left(\frac{2\sqrt{abxe^{-p}}}{1-xe^{-p}}\right)$
22	$\frac{x^{[t]}}{[t]!}L_{n}^{[t]-n}(a)$	$\frac{1-e^{-p}}{n!\,p}(xe^{-p}-a)^{n}\exp(xe^{-p})$
23	$\frac{x^{[t]}}{[t]!}L_{n}^{[t]+\alpha}(a)$	$\frac{1-e^{-p}}{p}\exp(xe^{-p})L_{n}^{\alpha}(a-xe^{-p})$
24	$x^{[t]}L_{[t]}^{\alpha-[t]}(a)$	$\frac{1-e^{-p}}{p}(1+xe^{-p})^{\alpha}\exp(-axe^{-p})$
25	$\frac{x^{[t]}}{[t]!}H_{[t]+n}(a)$	$\frac{1-e^{-p}}{p}\exp(2axe^{-p}-x^{2}e^{-2p})H_{n}(a-xe^{-p})$
26	$\frac{x^{2[t]}}{(2[t])!}H_{2[t]}(a)$	$\frac{1-e^{-p}}{p}\exp(-x^{2}e^{-p})\operatorname{ch}(2axe^{-p/2})$
27	$\frac{x^{2[t]+1}}{(2[t]+1)!}H_{2[t]+1}(a)$	$\frac{2}{p}\operatorname{sh}\frac{p}{2}\exp(-x^{2}e^{-p})\operatorname{sh}(2axe^{-p/2})$

TABLE 1 (continued)...

No.	$f([t])$	$F(p)$
28	$\frac{x^{2[t]+1}}{(2[t]+1)!} H_{2[t]}(a)$	$\frac{\sqrt{\pi}}{2p} e^{a^2} \operatorname{sh}\frac{p}{2}[\operatorname{erf}(a+xe^{-p/2})-\operatorname{erf}(a-xe^{-p/2})]$
29	$x^{[t]} C^{\nu}_{[t]}(a)$	$\frac{1-e^{-p}}{p}(1-2axe^{-p}+x^2e^{-2p})^{-\nu}, \quad \lvert xe^{-p}\rvert<1$
30	$\frac{x^{[t]}}{[t]!} B_{[t]}$	$\frac{1-e^{-p}}{p}\,\frac{xe^{-p}}{2}\left(\operatorname{cth}\frac{xe^{-p}}{2}-1\right), \quad \lvert xe^{-p}\rvert<2\pi$
31	$\frac{x^{2[t]}}{(2[t])!} B_{2[t]}$	$\frac{1-e^{-p}}{p}\,\frac{xe^{-p/2}}{2}\operatorname{cth}\frac{xe^{-p/2}}{2}, \quad \lvert xe^{-p/2}\rvert<2\pi$
32	$\frac{x^{[t]}}{[t]!} B_{[t]}(a)$	$\frac{1-e^{-p}}{p} xe^{-p}\exp(axe^{-p})[\exp(xe^{-p}-1)-1]^{-1}$
33	$\frac{x^{[t]}}{[t]!} E_{[t]}$	$\frac{1-e^{-p}}{p}\operatorname{sech}(xe^{-p}), \quad \lvert xe^{-p}\rvert<2\pi$
34	$\frac{x^{2[t]}}{(2[t])!} E_{2[t]}$	$\frac{1-e^{-p}}{p}\operatorname{sech}(xe^{-p/2}), \quad \lvert xe^{-p/2}\rvert<2\pi$
35	$\frac{x^{[t]}}{[t]!} E_{[t]}(a)$	$2\,\frac{1-e^{-p}}{p}\exp(axe^{-p})[\exp(xe^{-p})+1]^{-1}$
36	$\frac{x^{[t]}}{([t]+n)!} P^n_{[t]}(a)$	$(-1)^n\frac{1-e^{-p}}{p}\exp(axe^{-p}) J_n(xe^{-p}\sqrt{1-a^2})$
37	$\frac{x^{[t]}}{([t]+2n)!} P^n_{[t]+n}(a)$	$\frac{1-e^{-p}}{p}(xe^{-p})^{-n}\exp(axe^{-p}) I_n(xe^{-p}\sqrt{a^2-1})$
38	$\frac{(a)_{[t]}}{[t]!\,(b)_{[t]}} x^{[t]}{}_1F_1(a;\ b+[t];\ y)$	$\frac{1-e^{-p}}{p}\Phi_2(a,\ a;\ b;\ y,\ xe^{-p})$
39	$\frac{(b-a)_{[t]}}{[t]!\,(b)_{[t]}} x^{[t]}{}_1F_1(a;\ b+[t];\ y)$	$\frac{1-e^{-p}}{p}\exp(xe^{-p}){}_1F_1(a;\ b;\ y-xe^{-p})$
40	$\frac{a_{[t]}}{[t]!\,(c)_{[t]}} x^{[t]}{}_2F_1(a,\ b;\ c+[t];\ y)$	$\frac{1-e^{-p}}{p}\Xi_1(a,\ a,\ b;\ c;\ y,\ xe^{-p})$
41	$\frac{(a')_{[t]}(b')_{[t]}}{[t]!\,(c)_{[t]}} x^{[t]}{}_2F_1\times(a,\ b;\ c+[t];\ y)$	$\frac{1-e^{-p}}{p}F_3(a,\ a',\ b,\ b';\ y,\ xe^{-p}), \quad \lvert xe^{-p}\rvert<1, \quad \lvert y\rvert<1$
42	$\frac{(a)_{[t]}(b')_{[t]}}{[t]!\,(c')_{[t]}} x^{[t]}{}_2F_1(a+[t],\ b;\ c;\ y)$	$\frac{1-e^{-p}}{p}F_2(a,\ b,\ b';\ c,\ c';\ y,\ xe^{-p}), \quad \lvert xe^{-p}\rvert+\lvert y\rvert<1$
43	$\frac{(a)_{[t]}(b')_{[t]}}{[t]!\,(c)_{[t]}} x^{[t]}{}_2F_1(a+[t],\ b;\ c+[t];\ y)$	$\frac{1-e^{-p}}{p}F_1(a,\ b,\ b';\ c;\ y,\ xe^{-p}), \quad \lvert xe^{-p}\rvert<1, \lvert y\rvert<1$
44	$\frac{x^{[t]}}{(n-[t])!(\alpha+1)_{[t]}} L^{\alpha}_{[t]}(a)$, $t<n+1$; 0, $t>n+1$	$\frac{1-e^{-p}}{(\alpha+1)_n\,p}(xe^{-p}+1)^n L^{\alpha}_n\left(\frac{ax}{x+e^p}\right)$
45	$\frac{x^{[t]}}{(n-[t])!} L^{n-2[t]}_{[t]}(a)$, $t<n+1$; 0, $t>n+1$	$\frac{1-e^{-p}}{n!\,p}(-xe^{-p})^{n/2} H_n\left(\frac{1-axe^{-p}}{2\sqrt{-xe^{-p}}}\right)$

TABLE 1 (continued)...

No.	$f([t])$	$F(p)$
46	$\binom{n}{[t]} x^{[t]} H_{[t]}(a)$, $t<n+1$; 0, $t>n+1$	$\frac{1-e^{-p}}{p}(xe^{-p})^n H_n\left(a+\frac{e^p}{2x}\right)$
47	$\binom{n}{[t]} x^{[t]} H_{[t]}(a) H_{n-[t]}(b)$, $t<n+1$; 0, $t>n+1$	$\frac{1-e^{-p}}{p}(x^2e^{-2p}+1)^{n/2} H_n\left(\frac{axe^{-p}+b}{\sqrt{x^2e^{-2p}+1}}\right)$
48	$\frac{x^{[t]}}{(n-[t])!\,(2\nu)_{[t]}} C^{\nu}_{[t]}(a)$, $t<n+1$; 0, $t>n+1$	$\frac{1-e^{-p}}{(2\nu)_n p}(1-2axe^{-p}+x^2e^{-2p})^{n/2} \times$ $\times\, C^{\nu}_{n}\left(\frac{1-axe^{-p}}{\sqrt{1-2axe^{-p}+x^2e^{-2p}}}\right)$
49	$\binom{n}{[t]} x^{[t]} B_{[t]}(a)$, $t<n+1$; 0, $t>n+1$	$\frac{1-e^{-p}}{p}(xe^{-p})^n B_n\left(a+\frac{e^p}{x}\right)$
50	$\binom{n}{[t]} x^{[t]} E_{[t]}(a)$, $t<n+1$; 0, $t>n+1$	$\frac{1-e^{-p}}{p}(xe^{-p})^n E_n\left(a+\frac{e^p}{x}\right)$
51	$\binom{n}{[t]} x^{[t]}\, {}_{p+1}F_q(-[t],$ $(a_p);\, b_q;\, y)$, $t<n+1$; 0, $t>n+1$	$\frac{1-e^{-p}}{p}(xe^{-p}+1)^n_{p+1} F_q\left(-n,\,(a_p);\,(b_q);\,\frac{xy}{e^p+x}\right)$

LITERATURE CITED

1. V. A. Ditkin and A. P. Prudnikov, Integral Transforms and Operational Calculus [in Russian], Moscow (1974).
2. V. A. Ditkin and A. P. Prudnikov, Operational Calculus [in Russian], Moscow (1975).
3. Yu. A. Brychkov and A. P. Prudnikov, Inzh.-Fiz. Zh., 52, No. 6, 999-1003 (1987).
4. V. A. Ditkin and A. P. Prudnikov, Operational Calculus Handbook [in Russian], Moscow (1965).
5. A. P. Prudnikov, Yu. A. Brychkov, and O. I. Marichev, Integrals and Series. Elementary Functions [in Russian], Moscow (1981).
6. A. P. Prudnikov, Yu. A. Brychkov, and O. I. Marichev, Integrals and Series. Special Functions [in Russian], Moscow (1983).
7. A. P. Prudnikov, Yu. A. Brychkov, and O. I. Marichev, Integrals and Series. Additional Chapters [in Russian], Moscow (1986).